



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

360. Proposed by CHARLES C. GROVE, Columbia University, New York.

A bridge club of 28 members has 27 meetings. There are 7 tables with 4 members at each table. Can the players be so arranged that at the end of the season (27 meetings) each member will have played *with* every other member one game and *against* every other member two games, one game meaning one meeting; and how?

Solution by the PROPOSER.

The number of combinations of 28 people two at a time, ${}^{28}C_2$, is 14 times 27; of which each individual appears in ${}^{27}C_1$ or 27.

Thus each member plays *once with* each other member. At each of the 27 meetings 14 combinations of two each are playing, and so in the season all the possible combinations will be used, 14×27 . Further, at each meeting any member is playing *with* one member and *against* two other members. Since he thus goes around the membership *once with* each he will be going around *twice against* each.

The method of arranging the players can be seen from the following:

1, 2	2, 3	3, 4	4, 5	...	26, 27	27, 28
1, 3	2, 4	3, 5	4, 6	...	26, 28	
1, 4	2, 5	3, 6				
1, 5			4, 28			
1, 6		3, 28				
	2, 28					
1, 28						

where the numbers represent the different members of the club.

At the various meetings the tables must be filled from the above combinations.

Meeting	Table I.	II.	III.	IV.	V.	VI.	VII.
	1	5	9				
First	3 4	7 8	11 12	.	.	.	.
	2	6	10				
	1	5	9				
Second	2 4	6 8	10 12	.	.	.	.
	3	7	11				

	1	5	9	
Third	2 3	6 7	10 11	
	4	8	12	

All the combinations involving 1, 2, 3, 4 only have been used, and thereby 1 has played *with* each 2, 3, 4 once and twice against each. Since the players are identically involved, *mutatis mutandis*, all the other players will have a similar experience.

361. Proposed by C. E. GITHENS, Ph. D., Wheeling, W. Va.

Find three integral values for $[-10+9\sqrt{-3}]^{\frac{1}{3}}+[-10-9\sqrt{-3}]^{\frac{1}{3}}$. A solution not involving a cubic is desired.

I. Solution by the PROPOSER.

I. Put $(-10+9\sqrt{-3})^{\frac{1}{3}}+(-10-9\sqrt{-3})^{\frac{1}{3}}=\frac{1}{2}[(-80+72\sqrt{-3})^{\frac{1}{3}}+(-80-72\sqrt{-3})^{\frac{1}{3}}]$.

Let $(-80+72\sqrt{-3})^{\frac{1}{3}}=\sqrt[3]{x}+\sqrt[3]{y}$ and $(-80-72\sqrt{-3})^{\frac{1}{3}}=\sqrt[3]{x}-\sqrt[3]{y}$.

Then, $[6400-(-15552)]^{\frac{1}{3}}=x-y=28$, and, therefore,

$(-80+72\sqrt{-3})^{\frac{1}{3}}=\sqrt[3]{x+28}+\sqrt[3]{x}$. Hence, by raising both sides to the third power, $(4x+28)\sqrt[3]{x+28}+(4x+84)\sqrt[3]{x}=-80+72\sqrt{-3}$.

Put $4x+28\sqrt[3]{x+28}=-80$, and $4x+84\sqrt[3]{x}=72\sqrt{-3}$.

Let $4x+28=-80$ or factor of -80 ; -40 , -20 , -16 , -10 , -9 , -8 , -5 , -4 , -2 , and

$4x+84=72\sqrt{-3}$ or factor of 72 ; 36 , 24 , 18 , 12 , 9 , 8 , 6 , 4 , 3 , 2 .

Subtracting, $-56=-20-(36)$.

$\therefore 4x+28=-20$ and $x=-12$.

$$[1] \quad \frac{1}{2}[(-80+72\sqrt{-3})^{\frac{1}{3}}+(-80-72\sqrt{-3})^{\frac{1}{3}}] \\ =\frac{1}{2}[(\sqrt[3]{x+28}+\sqrt[3]{x})+(\sqrt[3]{x+28}-\sqrt[3]{x})]=4, \text{ answer.}$$

II. Similarly with $(-80+72\sqrt{-3})^{\frac{1}{3}}=\sqrt[3]{x_0+28}-\sqrt[3]{x_0}$, in which the factors -80 and 24 added to eliminate the $4x$'s produces

$x_0=-27$ and $\frac{1}{2}[(\sqrt[3]{x_0+28}-\sqrt[3]{x_0})+\sqrt[3]{x_0+28}+\sqrt[3]{x_0}]=-1$, answer.

III. With $-\sqrt[3]{x_1+28}+\sqrt[3]{x_1}$ and -16 and 72 as factors as above, $x=-3$ and $\frac{1}{2}[-\sqrt[3]{x_1+28}+\sqrt[3]{x_1}+(-\sqrt[3]{x_1+28}-\sqrt[3]{x_1})]^{\frac{1}{3}}=-5$, answer.

IV. The root $-\sqrt[3]{x_2+28}-x_2$ produces no two factors of -80 and 72 whose difference or sum equals ± 56 ; hence it is not a root, which is as it should be, for the numerical equation is an example of the "irreducible case" in the Cardan solution of a cubic whose equation is $x^3-21x+20=0$.